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# Maple as a tool to understand marketing game for up to four companies

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**Abstract.** In business environment, advertisement is very important. Companies spent a lot of money to advertise their products. When there are several players which have the same product to market, the competition will be quite tough since the market size is finite. The amount of the money spent for advertisement will affect the sale of the product. The profit for each company will depend on the amount of money spent for advertisement. However, this will also depend on the money spent by other companies, making the determination of the optimum profit become difficult. For two and three companies, exact value of the optimum profit for each company can be found. However, the analysis for more than three companies is quite complicated so we must resort to numerical method. Examples for the cases of two, three and four companies are given. A company with the smallest effectiveness factor and gross profit will have the smallest optimum profit. Maple has been widely used to derive formulas and to solve the case studies in this paper.

## 1. Introduction

In business environment, a market for a product may be catered by only one supplier or producer (monopoly) or two (duopoly) or more suppliers (oligopoly). In a monopoly environment, there is no competition at all; people will just buy the product. In a duopoly environment, there are only two players which dictate the market. They often compete aggressively against each other. There may be other players but their roles are insignificant. Good examples of duopoly environment are Boeing and Airbus, Visa and MasterCard, and Pepsi and Coca Cola. The competition is long lasting; every competitor does its best to capture a bigger market share.

When there are more than two suppliers, people have more choices; they can choose the supplier which offer lower price, assuming that the quality of the product from each supplier is similar. In this case the role of advertisement (or ads for short) is very important. Companies spent a lot of money to advertise their products. As an example, food companies advertise their product aggressively in the television for the children. Other examples are hand phone and automobiles companies; we can see their aggressive advertisement in the media (television, newspapers and magazines). We can now buy almost everything online sites. In Indonesia this online market is mostly dictated by BukaLapak, Shopee,



Blibli.com and Tokopedia. In Malaysia the biggest shares of broadband telecommunication are among Celcom, Maxis and Digi.

In oligopoly environment, the number of players which dictate the market is usually not more than four although it can be more; there is no exact definition for the number of players to be oligopoly. However, other players only have a small slice of the market. There is an entry barrier for new companies to enter the same market. Monopoly power and profitability in oligopolistic industries depend in part on how the firms interact [1]. When there are several players which have the same product to market, the competition will be quite tough since the market size is finite. It is possible for the companies to collude to fix the price of the product they sell in order to have more stable supplies and thus stable profits. However, this action may not be legal and they can be penalized by the authority. This action may also fail because other companies which are not involved in the collusion may reduce their price; this will in turn trigger the price reduction in the oligopolistic players. An example of this collusion in the US airline industry is given by [1].

Pricing is important for a company, but most companies with market power have another important decision to make: how much to advertise. Since advertising a product will cost a company a lot, it must decide carefully how much it will spend for the advertisement and where it will advertise so it will get maximum profit. The decisions depend on the characteristics of demand for the company's product.

A company which does not advertise enough may loss in the competition. Even if a product is good, it must be well advertised to capture the market unless the company monopolizes the market. For the role of advertising in oligopolistic markets see [2].

Nowadays, digital advertising is very common; its role is increasingly important, even more than printed media such as newspapers and magazines. According to Nielsen Media report, money spent for advertisement for the first seven month of 2020 in Indonesia is around Rp 122 trillion. This amount is distributed as follows: radio Rp 0.6 trillion, printed media Rp 9.6 trillion, websites Rp 24.2 trillion and television Rp 88 trillion. So, it is obvious that advertising a product is very important for the continuation life of a company. This does not mean that every product needs to be advertised. Corn or soybean are clear examples since their market are definite.

The purpose of this paper is to determine the optimum profit when there are up to four companies competing for the same product. We will only discuss theoretical analysis without delving further to real situations since they will need a lot of research into companies' balance sheets. However, this analysis can be applied if data needed from the companies are available.

## 2. Case of two companies

Duopoly is a special case of oligopoly. In this example, there are two companies that spend  $X_i$  dollars ( $i = 1, 2$ ) on advertising to capture a market for a certain product. Each company's advertising effectiveness factor is  $a_i$  and  $\beta_i$  is the elasticity of the company's attraction to the market where  $0 < a_i, \beta_i \leq 1$ . The companies (competitors)' market shares are given by [3]

$$s_1 = \frac{a_1 X_1^{\beta_1}}{a_1 X_1^{\beta_1} + a_2 X_2^{\beta_2}}; \quad s_2 = \frac{a_2 X_2^{\beta_2}}{a_1 X_1^{\beta_1} + a_2 X_2^{\beta_2}} \quad (1)$$

The total market size is  $S$  (the number products that is expected to be sold in the market). It is a logical assumption that the market is split in proportion to the money spent on advertising. In actual situation a company may actually spend less but will get a bigger share in the market but we do not consider this. The sale for each company is then given by  $Ss_i$ :

$$\frac{S a_i X_i^{\beta_i}}{a_1 X_1^{\beta_1} + a_2 X_2^{\beta_2}}; \quad i = 1, 2 \quad (2)$$

For a special case, let the elasticity of the company's attraction to the market be 1. The sale for each company is then given by

$$\frac{Sa_iX_i}{a_1X_1 + a_2X_2}, \quad i = 1, 2 \quad (3)$$

Suppose that the gross profit for every product of each company is  $g_i$ ,  $i = 1, 2$ , so the profit  $P_i$  of the company  $i$  is obtained as:

$$P_i = \frac{g_iSa_iX_i}{a_1X_1 + a_2X_2} - X_i, \quad i = 1, 2 \quad (4)$$

The optimal advertising amounts  $X_1$ ,  $X_2$  and the profits  $P_1$ ,  $P_2$  can be obtained by computing each company's "reaction curve":

$$R_i = \frac{\partial P_i}{\partial X_i}, \quad i = 1, 2 \quad (5)$$

By letting  $R_1 = R_2 = 0$  in (5), we can find the values of  $X_1$  and  $X_2$ . Thus, the problem can be solved easily as follow. From

$$P_1 = \frac{g_1Sa_1X_1}{a_1X_1 + a_2X_2} - X_1 \quad (6)$$

Then we have:

$$R_1 = \frac{g_1Sa_1}{a_1X_1 + a_2X_2} - \frac{g_1Sa_1^2X_1}{(a_1X_1 + a_2X_2)^2} - 1 = 0$$

which can be simplified to

$$g_1Sa_1a_2X_2 - (a_1X_1 + a_2X_2)^2 = 0 \quad (7)$$

Similarly, from:

$$P_2 = \frac{g_2Sa_2X_2}{a_1X_1 + a_2X_2} - X_2 \quad (8)$$

and by setting  $R_2 = \partial P_2 / \partial X_2 = 0$  we have:

$$g_2Sa_1a_2X_1 - (a_1X_1 + a_2X_2)^2 = 0 \quad (9)$$

Equations (7) and (9) can be solved in terms of  $X_1$  and  $X_2$ . The results are:

$$X_1 = \frac{Sg_1^2g_2a_1a_2}{(a_1g_1 + a_2g_2)^2}; \quad X_2 = \frac{Sg_2^2g_1a_1a_2}{(a_1g_1 + a_2g_2)^2} \quad (10)$$

Substituting  $X_1$  and  $X_2$  in (6) and (8), respectively, we find the profit for each company to be

$$P_1 = \frac{Sa_1^2g_1^3}{(a_1g_1 + a_2g_2)^2}; \quad P_2 = \frac{Sa_2^2g_2^3}{(a_1g_1 + a_2g_2)^2} \quad (11)$$

For numerical values, say that  $S = 10000$ ,  $a_1 = 0.85$ ,  $a_2 = 0.75$ ,  $g_1 = \$7$  and  $g_2 = \$8$ . We have:

Company 1:  $X_{1op} = \$17500$ ,  $P_{1Op} = \$17354$

Company 2:  $X_{2op} = \$20000$ ,  $P_{2Op} = \$20168$

The calculation is rounded to the nearest integer.

We can easily solve the problem using Maple as shown through an example in [4]. Maple is a symbolic computation language so it can solve the problem symbolically, not only numerically as opposed to many languages. Apart from Maple, we can also use Mathematica, another very powerful symbolic computation language. One advantage of Maple and Mathematica is that we can derive formulas we need; see [5] for examples.

### 3. Case of three companies

If there are three companies in the market, we can then write the profit for each company as follow:

$$P_1 = \frac{g_1 S a_1 X_1}{a_1 X_1 + a_2 X_2 + a_3 X_3} - X_1$$

$$P_2 = \frac{g_2 S a_2 X_2}{a_1 X_1 + a_2 X_2 + a_3 X_3} - X_2$$

$$P_3 = \frac{g_3 S a_3 X_3}{a_1 X_1 + a_2 X_2 + a_3 X_3} - X_3$$

Taking the partial derivative  $\partial P_i / \partial X_i = 0$  we then have after some simplification that

$$g_1 S a_1 a_2 X_2 + g_1 S a_1 a_3 X_3 - (a_1 X_1 + a_2 X_2 + a_3 X_3)^2 = 0$$

$$g_2 S a_1 a_2 X_1 + g_2 S a_2 a_3 X_3 - (a_1 X_1 + a_2 X_2 + a_3 X_3)^2 = 0$$

$$g_3 S a_1 a_3 X_1 + g_3 S a_2 a_3 X_2 - (a_1 X_1 + a_2 X_2 + a_3 X_3)^2 = 0$$

Maple can solve this set of equations; however, the result is very complicated. For the sake of information, the optimum amounts are given as follow. Curiously, Maple will write  $X_1$ ,  $X_2$  and  $X_3$  in different order every time it is run. So, the user should edit the result accordingly to suit his or her need.

$$X_1 = \frac{2g_1 g_2 g_3 S a_2 a_3}{(a_1 g_1 + a_2 g_2 + a_3 g_3)^2} [a_1 g_1 (a_2 g_2 + a_3 g_3) - a_2 g_2 a_3 g_3]$$

$$X_2 = \frac{2g_1 g_2 g_3 S a_1 a_3}{(a_1 g_1 + a_2 g_2 + a_3 g_3)^2} [a_2 g_2 (a_1 g_1 + a_3 g_3) - a_1 g_1 a_3 g_3]$$

$$X_3 = \frac{2g_1 g_2 g_3 S a_1 a_2}{(a_1 g_1 + a_2 g_2 + a_3 g_3)^2} [a_3 g_3 (a_1 g_1 + a_2 g_2) - a_1 g_1 a_2 g_2]$$

See that the result is cyclical; we can find  $X_2$  and  $X_3$  by changing  $X_1$  cyclically.

The optimum profit for each company is then given by

$$P_1 = \frac{S g_1}{(a_1 g_1 + a_2 g_2 + a_3 g_3)^2} [a_1 g_1 (a_2 g_2 + a_3 g_3) - a_2 g_2 a_3 g_3]^2$$

$$P_2 = \frac{S g_2}{(a_1 g_1 + a_2 g_2 + a_3 g_3)^2} [a_2 g_2 (a_1 g_1 + a_3 g_3) - a_1 g_1 a_3 g_3]^2$$

$$P_3 = \frac{S g_3}{(a_1 g_1 + a_2 g_2 + a_3 g_3)^2} [a_3 g_3 (a_1 g_1 + a_2 g_2) - a_1 g_1 a_2 g_2]^2$$

respectively. The result is also cyclical.

Suppose a third company enter the same market as companies 1 and 2. Let the parameters for companies 1 and 2 be the same as in the previous example and for company 3 we have  $a_3 = 0.72$  and  $g_3 = \$6.7$ . We have the following data:  $S = 10000$ ,  $a_1 = 0.85$ ,  $a_2 = 0.75$ ,  $a_3 = 0.72$ ,  $g_1 = \$7$ ,  $g_2 = \$8$  and  $g_3 = \$6.7$ . We then have:

Company 1:  $X_{1op} = \$16490$ ,  $P_{1Op} = \$10101$

Company 2:  $X_{2op} = \$18943$ ,  $P_{2Op} = \$11860$

Company 3:  $X_{3op} = \$12049$ ,  $P_{3Op} = \$3704$

We can see that when company 3 enter the market, the profits of companies 2 and 3 drop significantly. The profit for company 2 drops about 41.8% while the profit for company 2 drops about 41.2%. Even though the profit for company 3 is still small compared to profits companies 2 and 3, the company 3 has reduced other companies' profits.

#### 4. Case of four companies

We can extend the analysis using more companies. Unfortunately, the formulae for the optimal advertising amount and the profit for each company become very complicated. So, it is much easier if we just use numerical values. Say that a new company enters the market in which  $a_4 = 0.65$  and  $g_4 = 5.5$ . Using Maple, we get the following results:

Company 1:  $X_{1op} = \$16414$ ,  $P_{1Op} = \$9866$

Company 2:  $X_{2op} = \$18860$ ,  $P_{2Op} = \$11591$

Company 3:  $X_{3op} = \$11853$ ,  $P_{3Op} = \$3533$

Company 4:  $X_{4op} = \$816$ ,  $P_{4Op} = \$12$

We can see that in a competitive market, the player with better effectiveness factor and gross profit will gain higher profit. Since company that has the smallest advertising effectiveness factor and also the smallest gross profit, its optimal advertising amount and profit are also the smallest. A big company can usually expect high order from the market when it spends more on advertising. On the other hand, a small company does not have a lot of money to spend on advertising so it cannot capture a bigger market. Some people may argue that there are examples when a smaller company can still enjoy a bigger market. The argument is valid but that will depend on the marketing strategy of the company. Here, we assume that advertising will help companies sell their products better.

#### 5. Maple as a tool for solving operations research problems

Maple is very useful for solving problem symbolically. This is an obvious advantage if we want to derive a formula and solve a problem for many inputs. There are many good books on Maple and its applications for solving problems available; see [5], [6] and [7], for example. However, its capability to solve numerical problems is also great. Maple also has a package for solving operations research problems such as linear programming and nonlinear programming; see [8], for example. However, the cases discussed in this paper must be solved by writing suitable programs. As the number of companies increases, the exact solutions become very complicated and may not be available for more than four companies. In this situation, we must resort to numerical methods.

Apart from Maple, MATLAB can also solve symbolically the problems discussed in this paper; see [6], for example. However, it is rather slow and the results displayed on the screen are not user friendly. It is good to derive the generalized forms using Maple first and then switch to MATLAB.

#### 6. Conclusion

Analysis has shown that optimum profit for each company in a marketing game depends not only on the effectiveness factor and gross profit of the company itself but also by other company in the game. The player with better effectiveness factor and gross profit will gain higher profit. When a new player enters the market, the competition becomes tougher. A company with a smaller effectiveness factor and gross profit will see its optimum profit reduced.

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